Inclusion of QED corrections in PDFs

The NNPDF4.0QED PDF set

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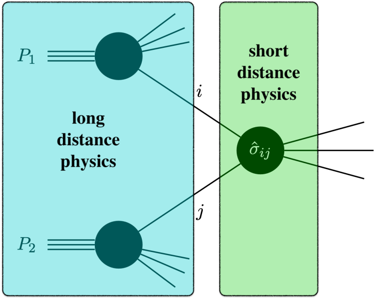
1. Introduction
2. Adding QED
3. Impact on phenomenology
4. Conclusions

What are PDFs?

- In high energy physics, cross sections are computed via



$$\underbrace{\sigma(Q^2)}_{\text{long distance physics}} = f_{1,i}(x_1, Q^2) \otimes f_{2,j}(x_2, Q^2) \otimes \underbrace{\hat{\sigma}_{ij}(x_1, x_2, Q^2)}_{\text{short distance physics}}$$

- x_i proton momentum fraction carried by the parton
- Q^2 hard scale of the process

- The **parton distribution functions** (PDFs) link the two blocks.
- Comparing theoretical predictions with experimental data we can **extract** $f_i(Q^2)$ for a given Q^2.

DGLAP evolution equation

- DGLAP gives the Q^2 dependence of the PDFs:

DGLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(\mu^2)\right) f_j(z,\mu^2) \quad i = q,\bar{q},g$$

$$P_{ij} = \underbrace{\underbrace{\underbrace{\alpha_s P_{ij}^{(0)}}_{\text{LO}} + \alpha_s^2 P_{ij}^{(1)}}_{\text{NLO}} + \alpha_s^3 P_{ij}^{(2)}}_{\text{NNLO}} + \ldots \quad \leftarrow \quad \text{Splitting functions}$$

Solution: $f_i(x,Q_0^2) \xrightarrow{\text{DGLAP}} f_i(x,Q^2) = E_{ij}(Q^2 \leftarrow Q_0^2) \otimes f_j(x,Q_0^2)$

- We can evolve f_i to all scales Q^2.

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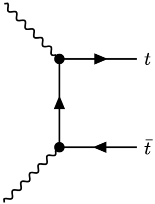
QED fit: Motivation

- $\mathcal{O}(\alpha_{\rm em}) \sim \mathcal{O}(\alpha_s^2) \sim \mathcal{O}(0.01) \implies$ percent correction
- At the moment no photon-induced (PI) contributions in theory predictions
- For example: $t\bar{t}$ PI starts at $\mathcal{O}(\alpha_s^0)$



- **We need to add QED corrections!**

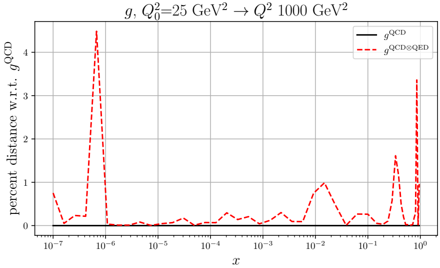
Corrections to DGLAP

- DGLAP has QED corrections

$$P_{ij}(\alpha_s) \rightarrow P_{ij}(\alpha_s, \alpha_{\text{em}}) = \overbrace{P_{ij}^{\text{QCD}}(\alpha_s)}^{\text{pure QCD terms}} + \overbrace{P_{ij}^{\text{QCD}\otimes\text{QED}}(\alpha_s, \alpha_{\text{em}})}^{\text{pure QED and QED}\otimes\text{QCD terms}}$$

$$P_{ij}^{\text{QCD}\otimes\text{QED}}(\alpha_s, \alpha_{\text{em}}) = \alpha_{\text{em}} P_{ij}^{(0,1)} + \alpha_s \alpha_{\text{em}} P_{ij}^{(1,1)} + \alpha_{\text{em}}^2 P_{ij}^{(0,2)}$$

- gluon couples in the same way to all quarks
- photon distinguishes up-like and down-like $\Longrightarrow$ more diffucult to diagonalize



Small correction to QCD PDFs evolution

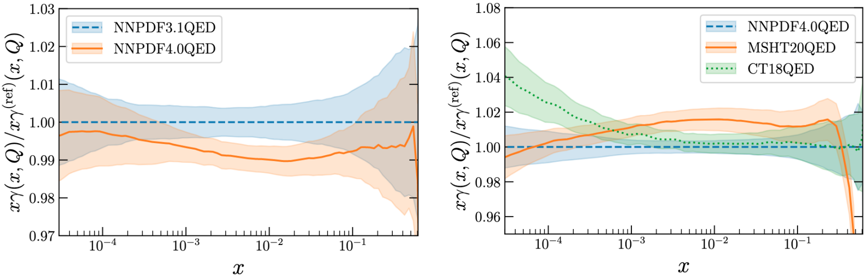
QED fit: Photon PDF

- We get a photon PDF $\gamma(x, Q^2)$
- It can be **computed**: LuxQED approach [Manohar, Nason, Salam, Zanderighi, 2016]

$$x\gamma(x,\mu^2) = \frac{1}{2\pi\alpha_{\rm em}(\mu^2)} \int_x^1 \left\{ \frac{dz}{z} \int_{\frac{m_p^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha_{\rm em}^2(Q^2) \left[-z^2 F_L(x/z, Q^2) + \left(zP_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) \right] - \alpha_{\rm em}^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\} + \mathcal{O}(\alpha_s\, \alpha_{\rm em}, \alpha_{\rm em}^2)$$

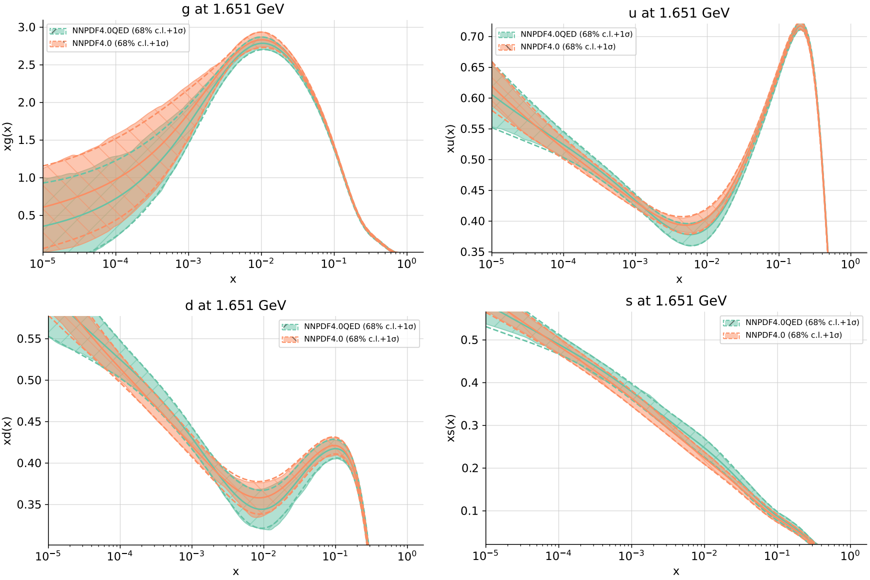
- $F_{2,L}$ are computed from QCD PDFs: $F_{2,L} = f \otimes C_{2,L}$
- $\gamma(x)$ modifies sum rules: $\int_0^1 dx\, x \left(\sum_{q,\bar{q}} q(x,Q^2) + g(x,Q^2) + \gamma(x,Q^2) \right) = 1$

QED fit: Comparison with other PDF sets



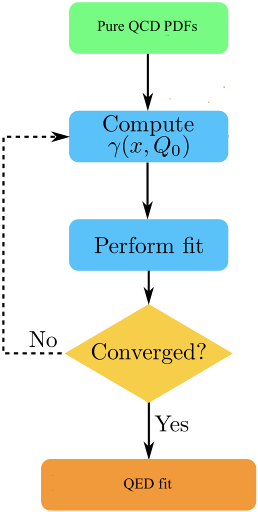
γ is compatible with the other QED PDF sets

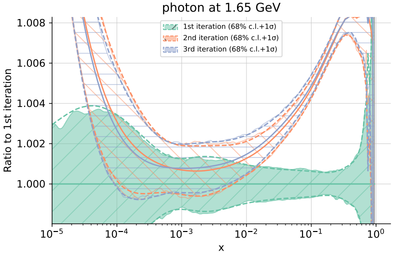
QED fit: Comparison with NNPDF4.0

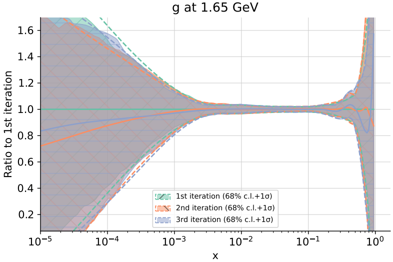


QED fit: Iteration

- γ depends on QCD PDFs but it changes them $\Longrightarrow$ we have to iterate







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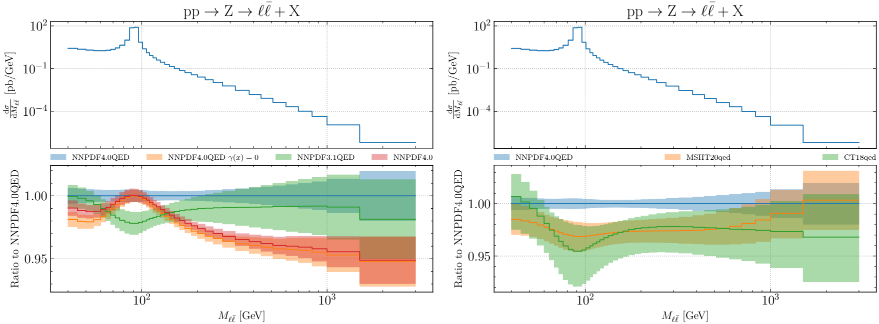
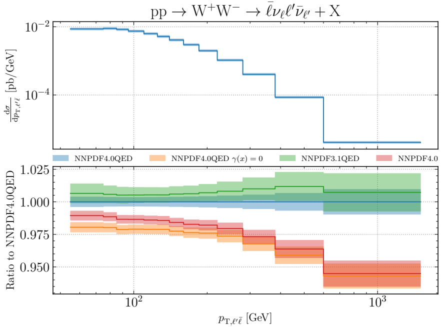
Phenomenology: inclusive Drell-Yan production

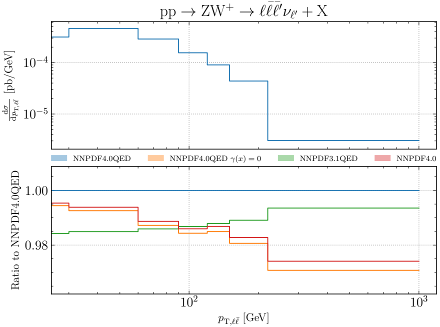


$\sqrt{s} = 14\,\mathrm{TeV}$, $M_{\ell\bar{\ell}}$ invariant mass of $\ell\bar{\ell}$

In the high $M_{\ell\bar{\ell}}$ region QED corrections are not negligible! $\mathcal{O}(5\%)$

Phenomenology: weak bosons pair production





5% correction

2-3% correction

Few words on a new pipeline

- NNPDF4.0: based on `APFEL`, `APFELgrid` and `APPLgrid`
- NNPDF4.0QED: based on new tools `EKO`, `YADISM` and `PineAPPL`

- It will be possible to include photon induced contribution in the theory predictions!

- It is possible to include QED corrections to PDF fits.
- The photon PDF is compatible with the most recent QED PDF sets.
- The quark and gluon PDFs are almost unchanged.
- There are processes in which QED gives a non negligible contribution.

Thanks for your attention!

5 Backup slides

Unified evolution basis

$$g$$

$$\gamma$$

$$\Sigma = \Sigma_u + \Sigma_d$$

$$\Sigma_\Delta = \frac{n_d}{n_u}\Sigma_u - \Sigma_d$$

$$V = V_u + V_d$$

$$V_\Delta = \frac{n_d}{n_u}V_u - V_d$$

$$T_3^d = d^+ - s^+$$

$$V_3^d = d^- - s^-$$

$$T_3^u = u^+ - c^+$$

$$V_3^u = u^- - c^-$$

$$T_8^d = d^+ + s^+ - 2b^+$$

$$V_8^d = d^- + s^- - 2b^-$$

$$T_8^u = u^+ + c^+ - 2t^+$$

$$V_8^u = u^- + c^- - 2t^-$$

$$\Sigma_u = \sum_{k=1}^{n_u} u_k^+,\ \Sigma_d = \sum_{k=1}^{n_d} d_k^+,\ V_u = \sum_{k=1}^{n_u} u_k^-,\ V_d = \sum_{k=1}^{n_d} d_k^-$$

DGLAP equations

- Singlet sector

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix} =$$

$$- \begin{pmatrix} \gamma_{gg} + \tilde{\gamma}_{gg} & \tilde{\gamma}_{g\gamma} & \gamma_{gq} + \langle \tilde{\gamma}_{gq} \rangle & \nu_u \tilde{\gamma}_{g\Delta q} \\ \tilde{\gamma}_{\gamma g} & \tilde{\gamma}_{\gamma\gamma} & \langle \tilde{\gamma}_{\gamma q} \rangle & \nu_u \tilde{\gamma}_{\gamma \Delta q} \\ 2n_f(\gamma_{qg} + \langle \tilde{\gamma}_{qg} \rangle) & 2n_f \langle \tilde{\gamma}_{q\gamma} \rangle & \gamma_{qq} + \langle \tilde{\gamma}_q^{\mathrm{ns},+} \rangle + \langle e_q^2 \rangle^2 \tilde{\gamma}_{\mathrm{ps}} & \nu_u \tilde{\gamma}_{\Delta q}^{\mathrm{ns},+} + \nu_u e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{\gamma}_{\mathrm{ps}} \\ 2n_f \nu_d \tilde{\gamma}_{\Delta q g} & 2n_f \nu_d \tilde{\gamma}_{\Delta q \gamma} & \nu_d \tilde{\gamma}_{\Delta q}^{\mathrm{ns},+} + \nu_d e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{\gamma}_{\mathrm{ps}} & \gamma_{\mathrm{ns},+} + \{ \tilde{\gamma}_q^{\mathrm{ns},+} \} + \nu_u \nu_d (e_{\Delta q}^2)^2 \tilde{\gamma}_{\mathrm{ps}} \end{pmatrix} \otimes \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix}$$

- Valence sector

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} V \\ V_\Delta \end{pmatrix} = - \begin{pmatrix} \gamma_{\mathrm{ns},V} + \langle \tilde{\gamma}_q^{\mathrm{ns},-} \rangle & \nu_u \tilde{\gamma}_{\Delta q}^{\mathrm{ns},-} \\ \nu_d \tilde{\gamma}_{\Delta q}^{\mathrm{ns},-} & \gamma_{\mathrm{ns}-} + \{ \tilde{\gamma}_q^{\mathrm{ns},-} \} \end{pmatrix} \otimes \begin{pmatrix} V \\ V_\Delta \end{pmatrix}$$

- Decoupled sector:

$$\mu^2 \frac{d}{d\mu^2} T_{3/8}^{u/d} = -(\gamma_{\mathrm{ns},+} + \tilde{\gamma}_{u/d}^{\mathrm{ns},+}) T_{3/8}^{u/d} ,$$

$$\mu^2 \frac{d}{d\mu^2} V_{3/8}^{u/d} = -(\gamma_{\mathrm{ns},-} + \tilde{\gamma}_{u/d}^{\mathrm{ns},-}) V_{3/8}^{u/d} .$$

Phenomenology: $t\bar{t}$