UNIVERSITÀ DEGLI STUDI DI MILANO

N3PDF
Machine Learning • PDFs • QCD

The inclusion of QED corrections in the NNPDF4.0 fitting framework

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on behalf of the **NNPDF** collaboration
Based on [2401.08749]

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NNPDF

Outline

PDFs fitting

How to add QED effects

Results

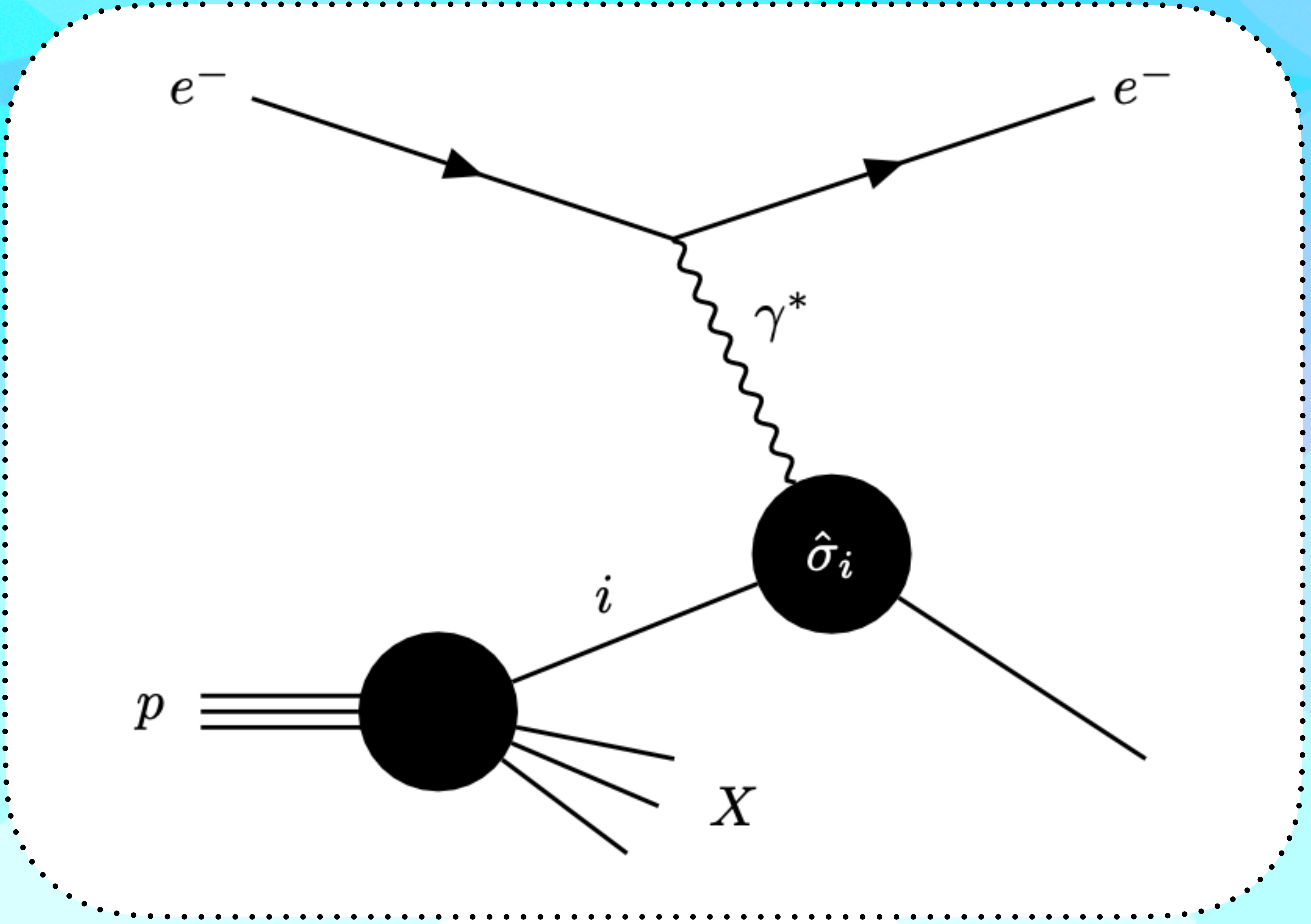
Impact on phenomenology

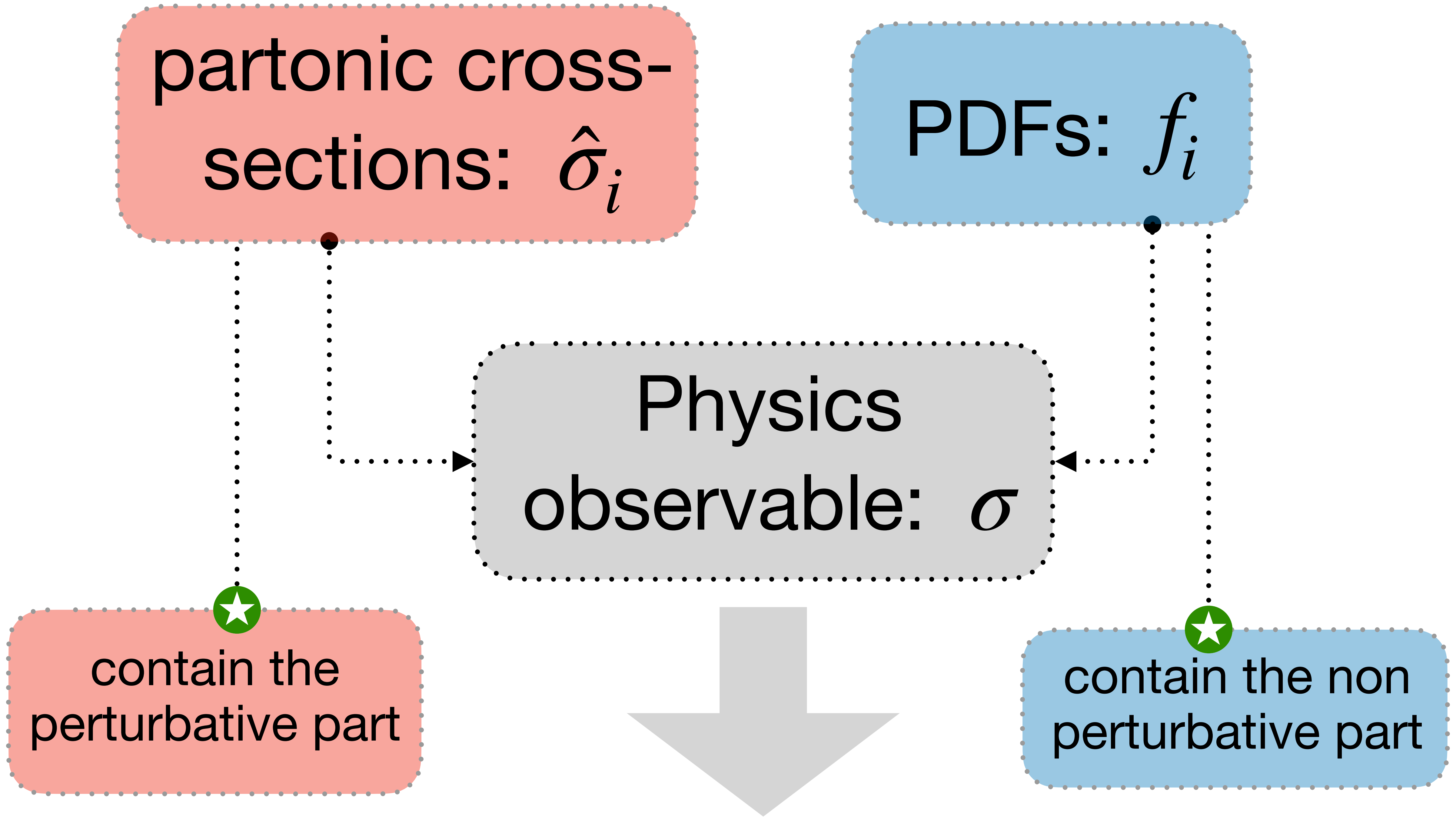
Summary and Outlook

PDFs fitting

- How do we compute observables in HEP?
- What are the PDFs?

Deep inelastic scattering (DIS)

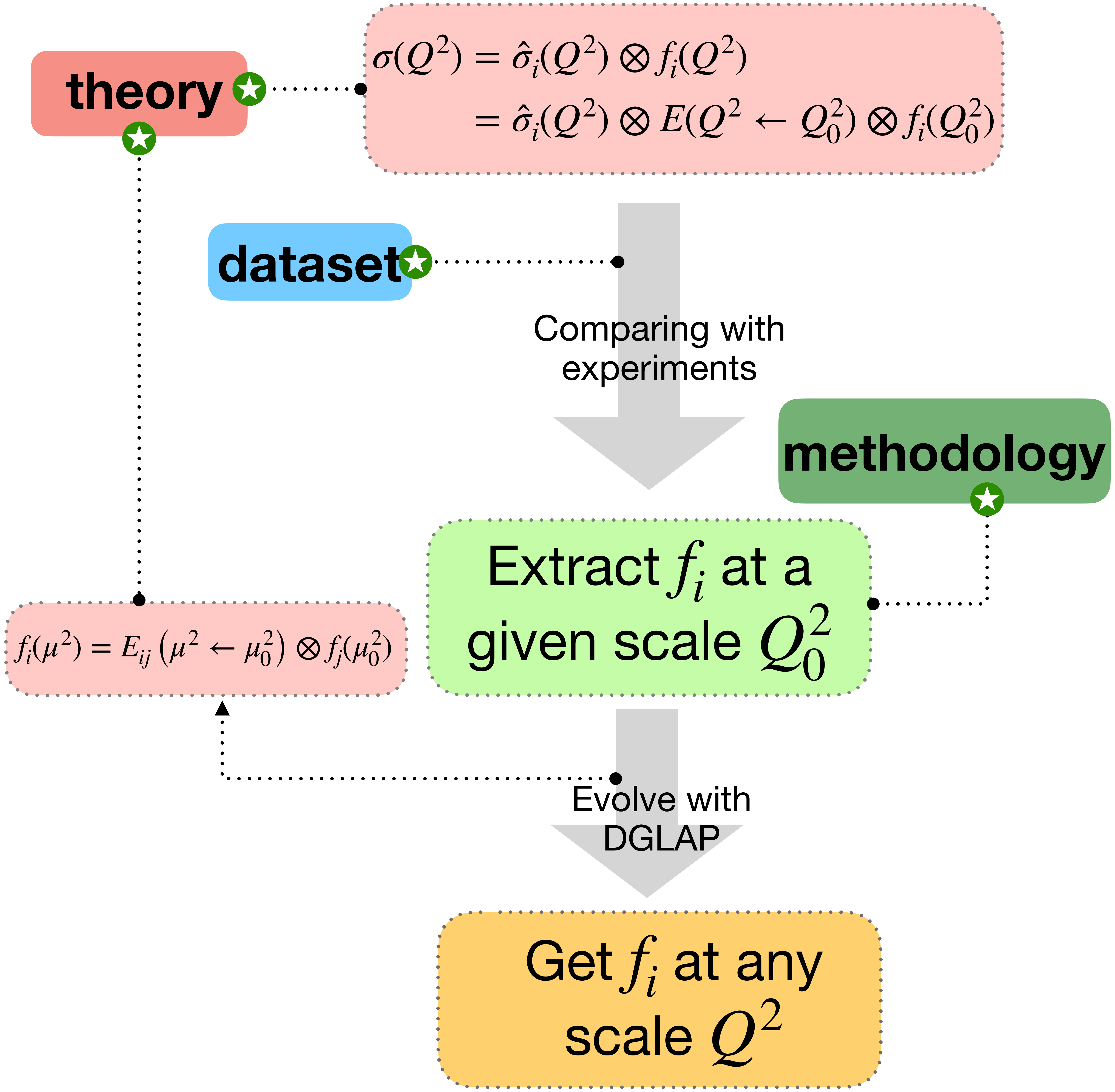




Factorization theorem

$$\sigma = \sum_i \hat{\sigma}_i \otimes f_i + \mathcal{O}(\Lambda^2/Q^2)$$

- How are the PDFs fitted?
- We have to define a **theory**
- We have to choose a **dataset**
- We have to choose a fitting **methodology**



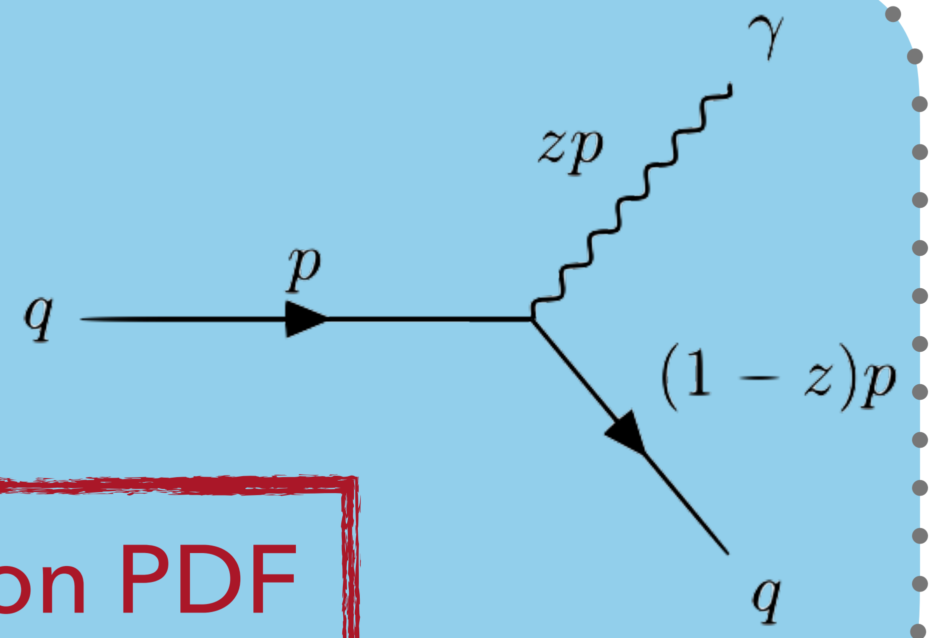
How to add QED effects

- Why do we want to add QED effects in PDFs?
- Are there cases in which they are not negligible?

$$\alpha \sim \mathcal{O}(\alpha_s^2) \sim \mathcal{O}(1\%)$$

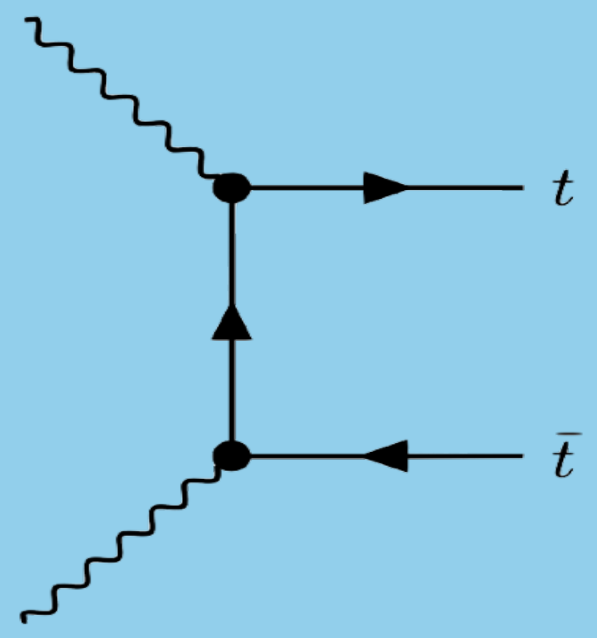
percent correction

quarks can emit photons



we get a photon PDF

For some processes we can't neglect photon induced contributions



starts at $\mathcal{O}(\alpha_s^0)$

- How is the photon PDF determined?
- LuxQED gives a constraint between the photon PDF and the QCD PDFs

LuxQED approach

$$x\gamma(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)}\int_x^1 \frac{dz}{z}\left\{\int_{\frac{m_p^2x^2}{1-z}}^{\frac{\mu^2}{1-z}}\frac{dQ^2}{Q^2}\alpha^2(Q^2)\left[\left(zP_{\gamma q}(z) + \frac{2x^2m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2F_L(x/z,Q^2)\right] - \alpha^2(\mu^2)z^2F_2(x/z,\mu^2)\right\}$$

$$F_{2,L} = \sum_i C_{2,L,i}\otimes f_i$$

It modifies the sum rules

$$\int_0^1 dx\, x\left(\Sigma(x,Q^2) + g(x,Q^2) + \gamma(x,Q^2)\right) = 1$$

- How are DGLAP equations in presence of QED corrections?
- The photon PDF mixes with the other PDFs through evolution

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \sum_{j=q,\bar{q},g,\gamma} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(\mu^2), \alpha(\mu^2)\right) f_j(z,\mu^2)$$

$$i = q, \bar{q}, g, \gamma$$

$$P_{ij}(\alpha_s, \alpha) = P_{ij}^{\mathrm{QCD}}(\alpha_s) + \tilde{P}_{ij}(\alpha_s, \alpha)$$

pure QCD terms

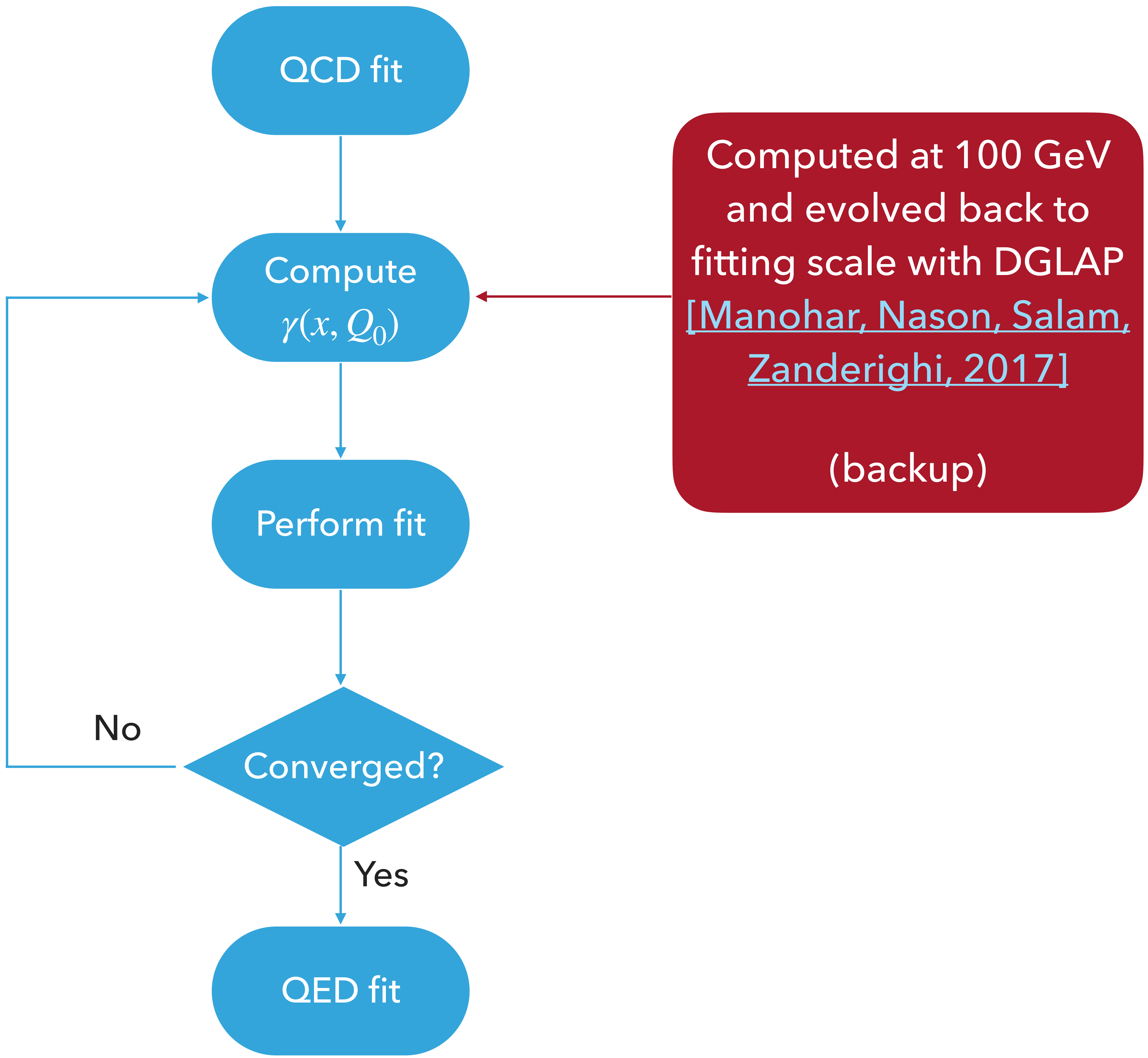
$$P_{ij}^{\mathrm{QCD}}(\alpha_s) = \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)} + \dots$$

pure QED and QCD⊗QED terms

$$\tilde{P}_{ij}(\alpha_s, \alpha) = \alpha P_{ij}^{(0,1)} + \alpha_s \alpha P_{ij}^{(1,1)} + \alpha^2 P_{ij}^{(0,2)} + \dots$$

The QED case is more difficult to solve than the pure QCD one (backup)

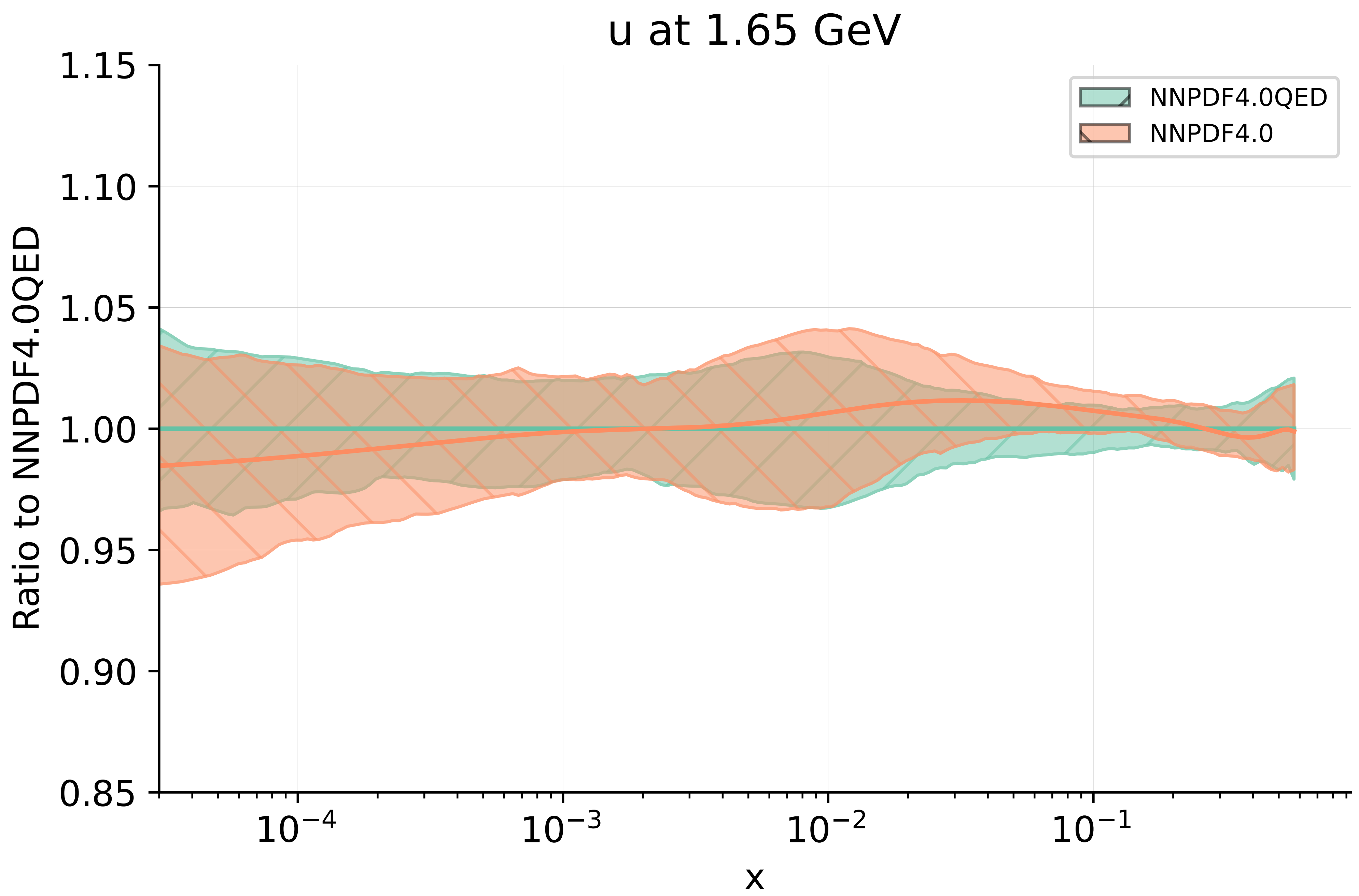
- What is the fitting methodology?
- LuxQED formula gives a constraint between γ and the other PDFs: **such constraint is implemented iteratively**

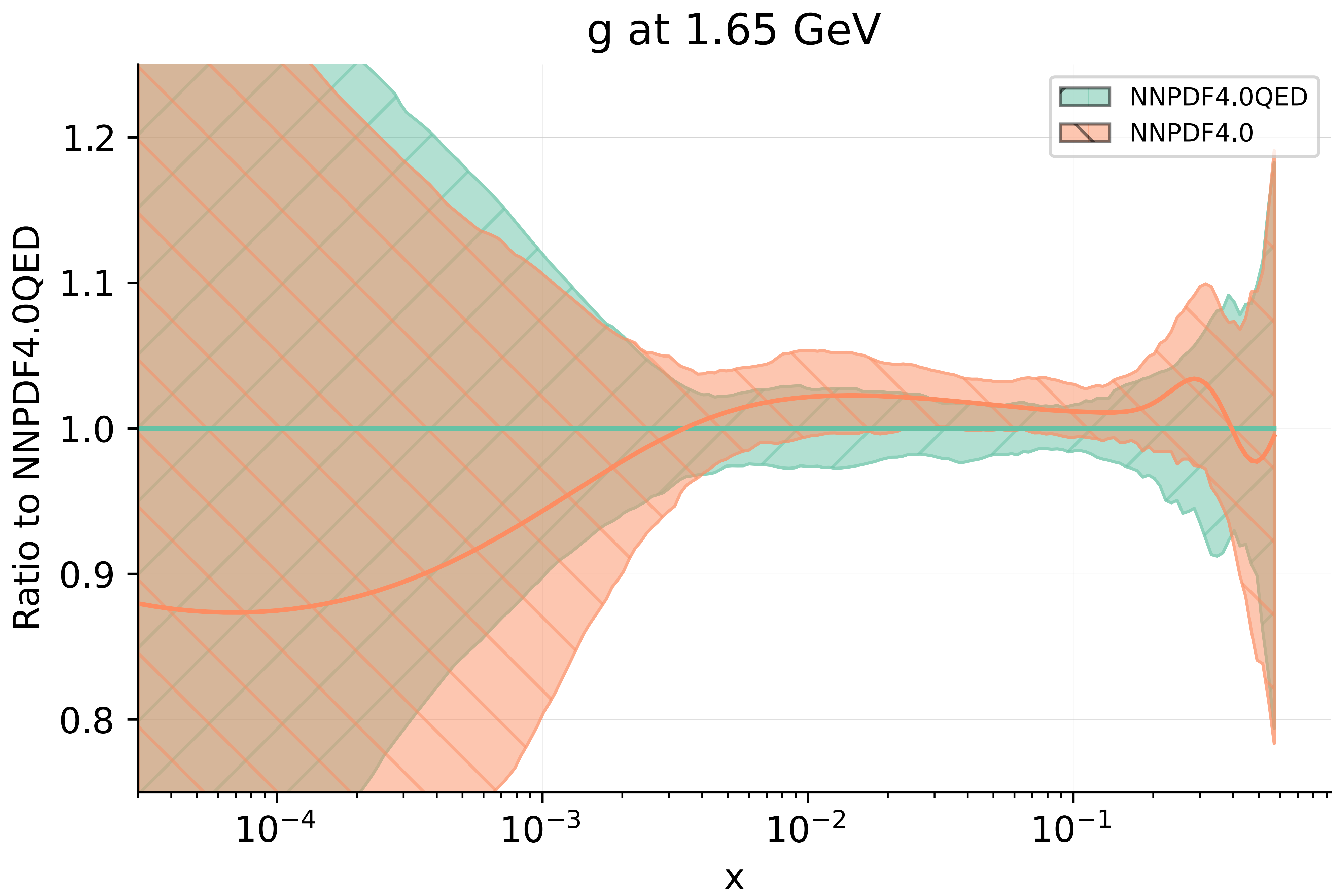


Results

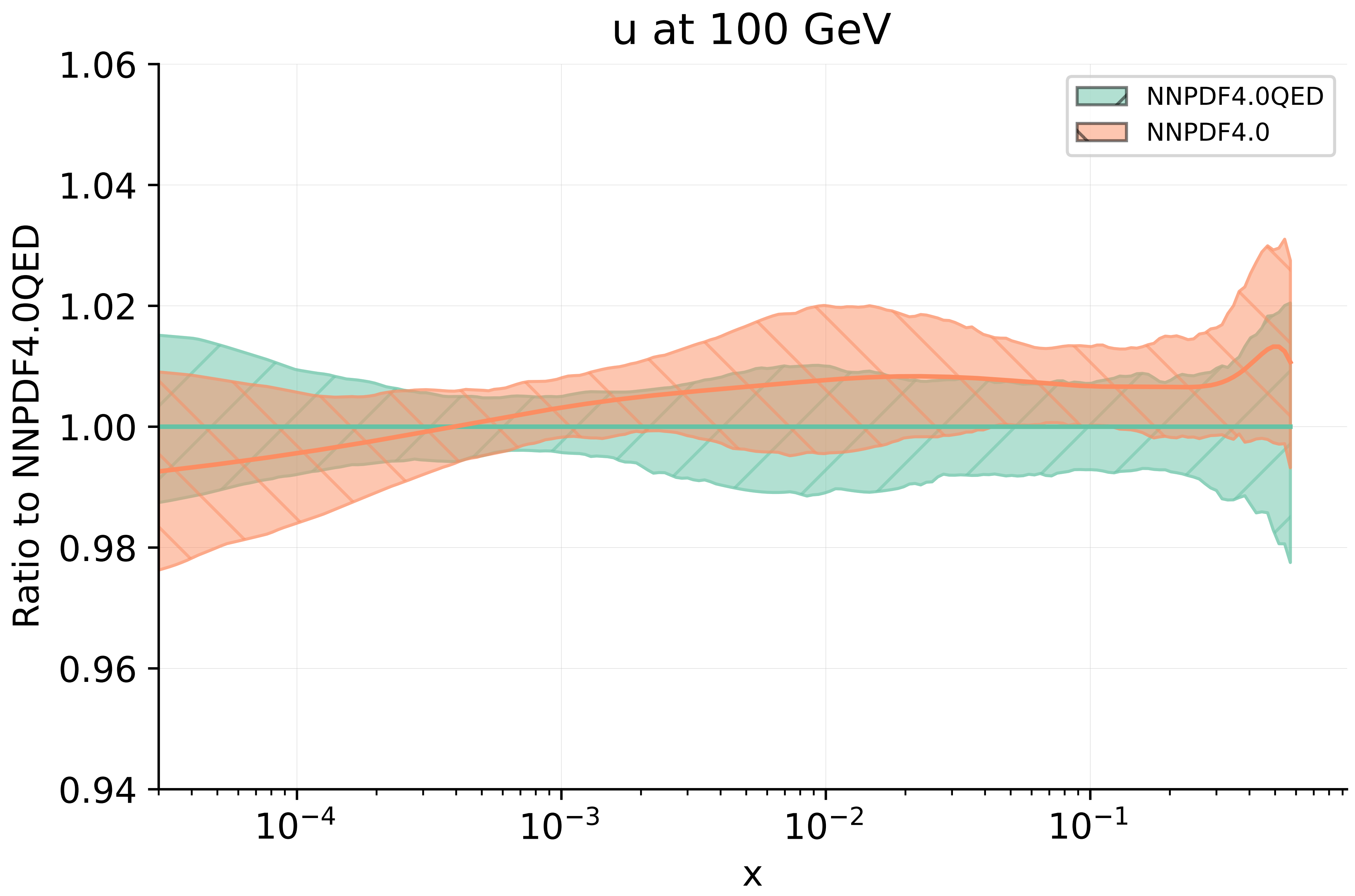
- PDFs fitting
- How to add QED effects
- **Results**
- Impact on phenomenology
- Summary and Outlook

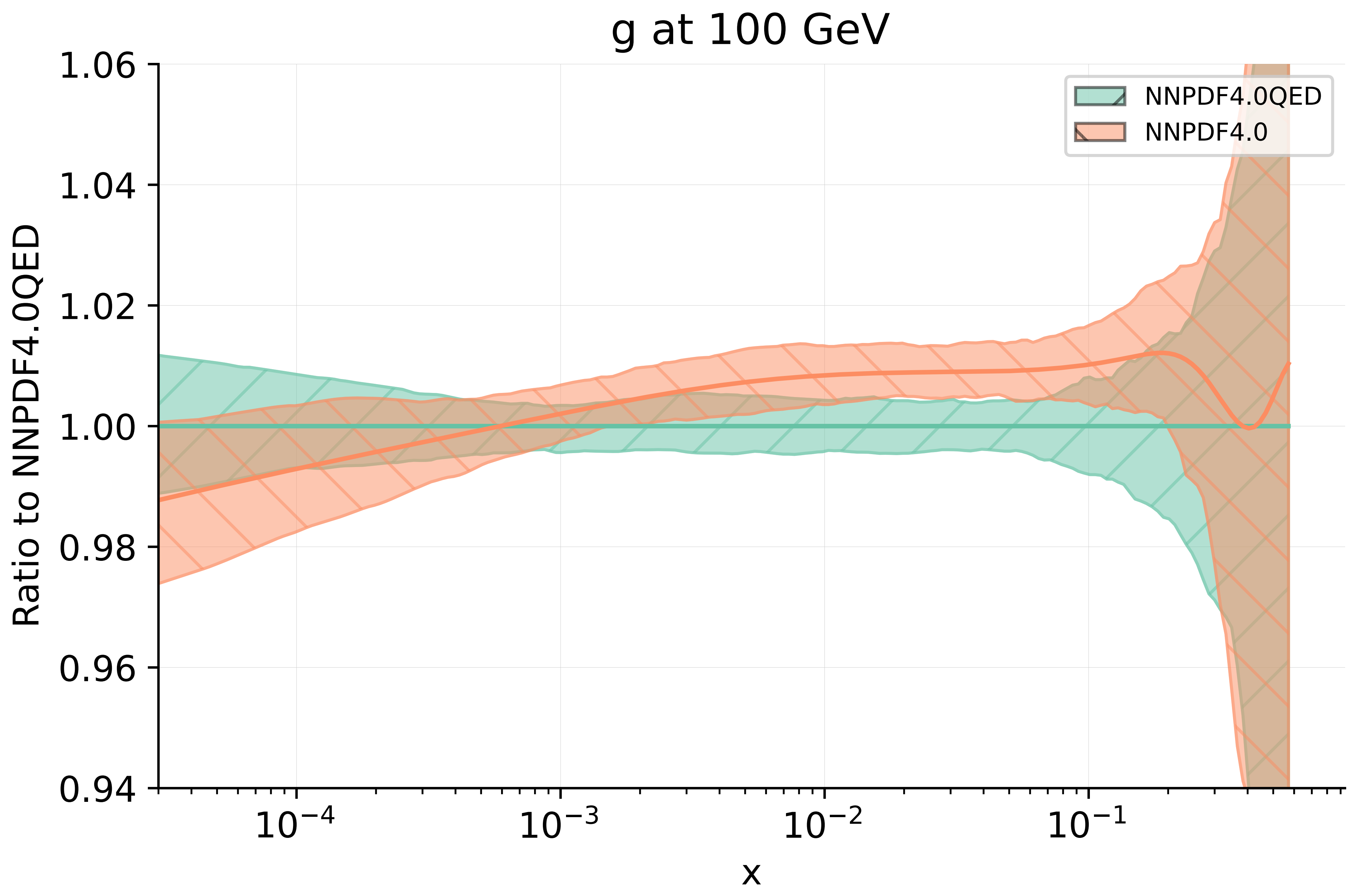
- Results at fitting scale
- Very small differences in the quarks and gluon



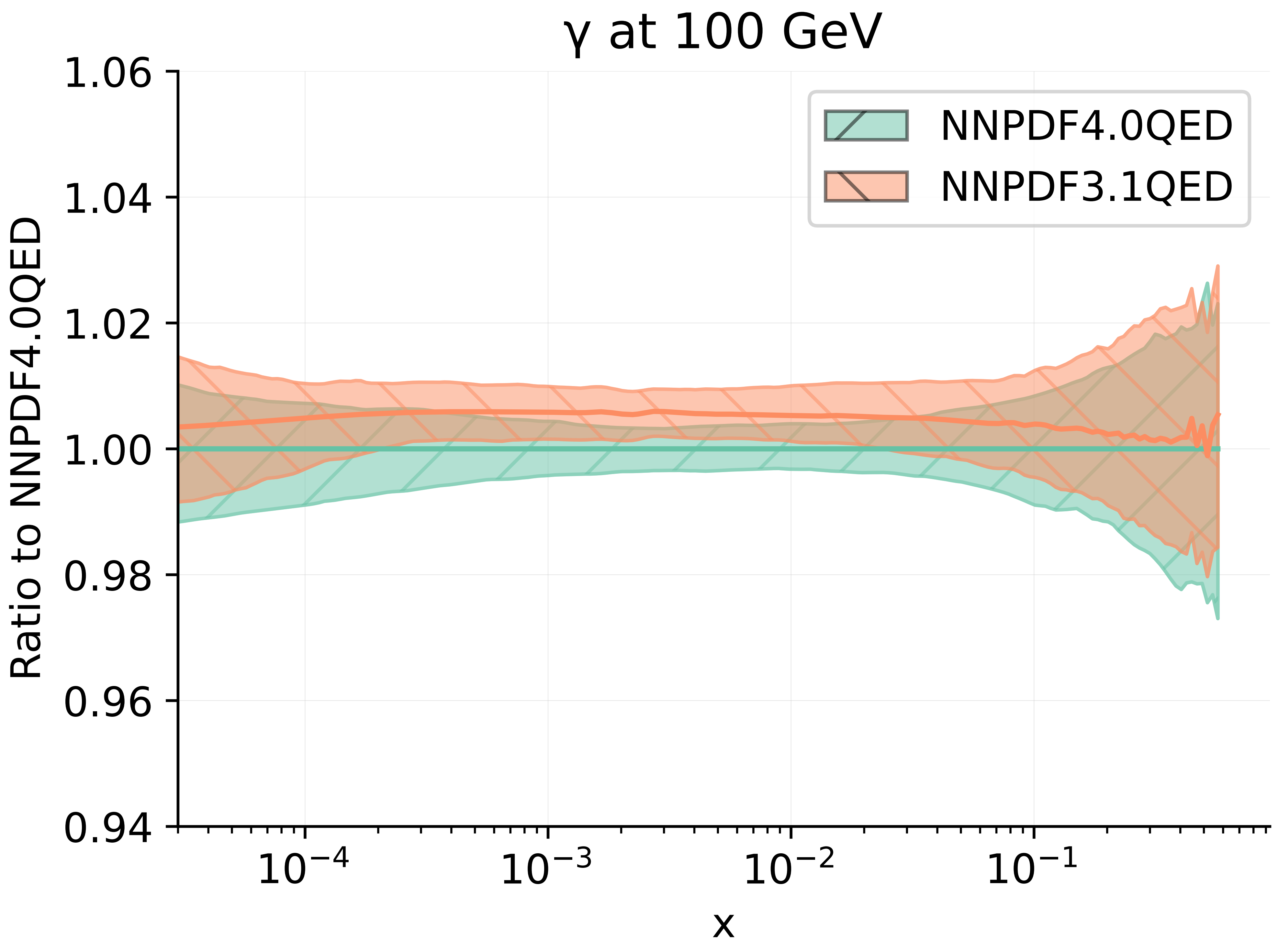


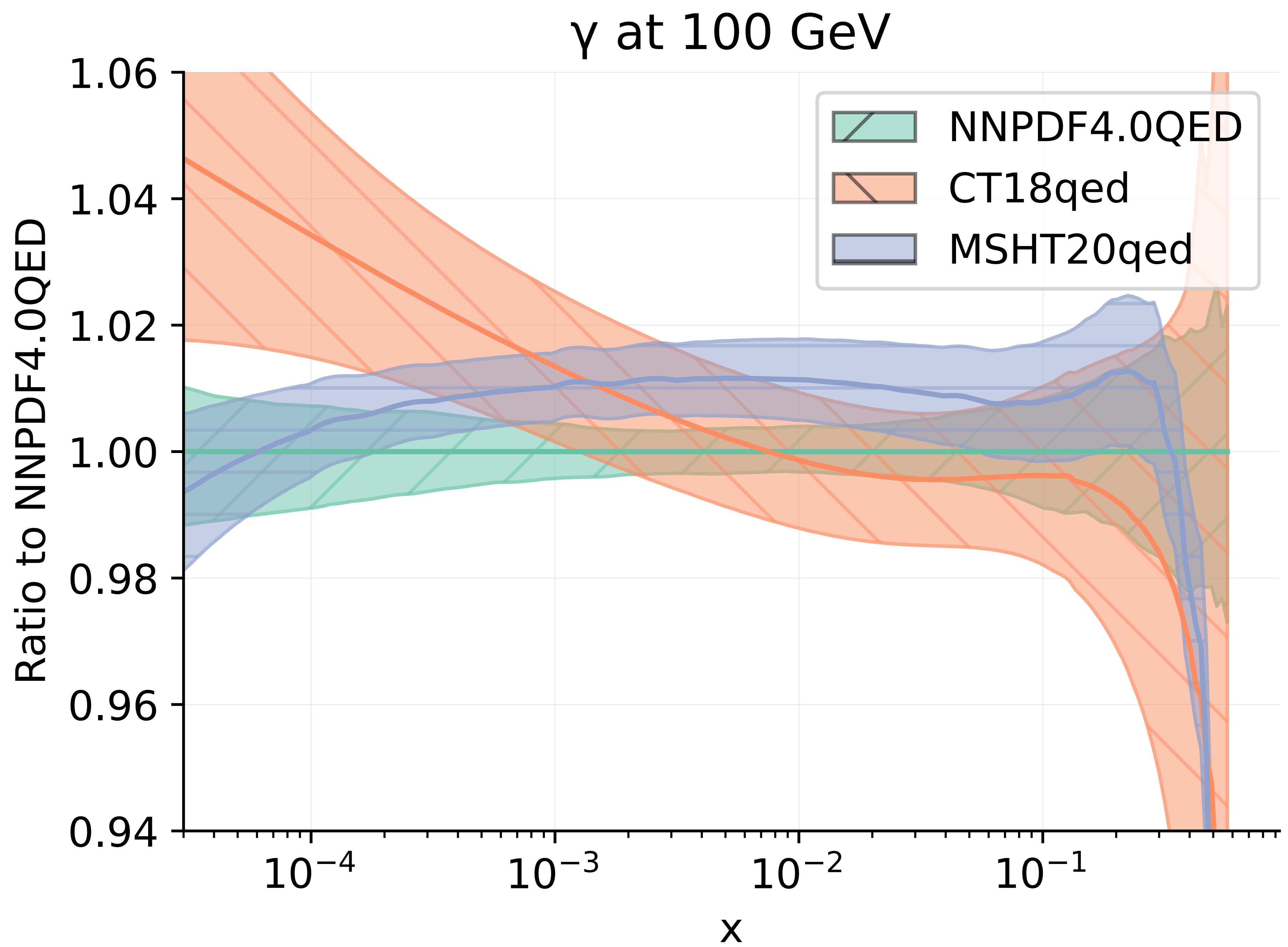
- Results at 100 GeV
- Difference grows due to the effect of the photon in the evolution





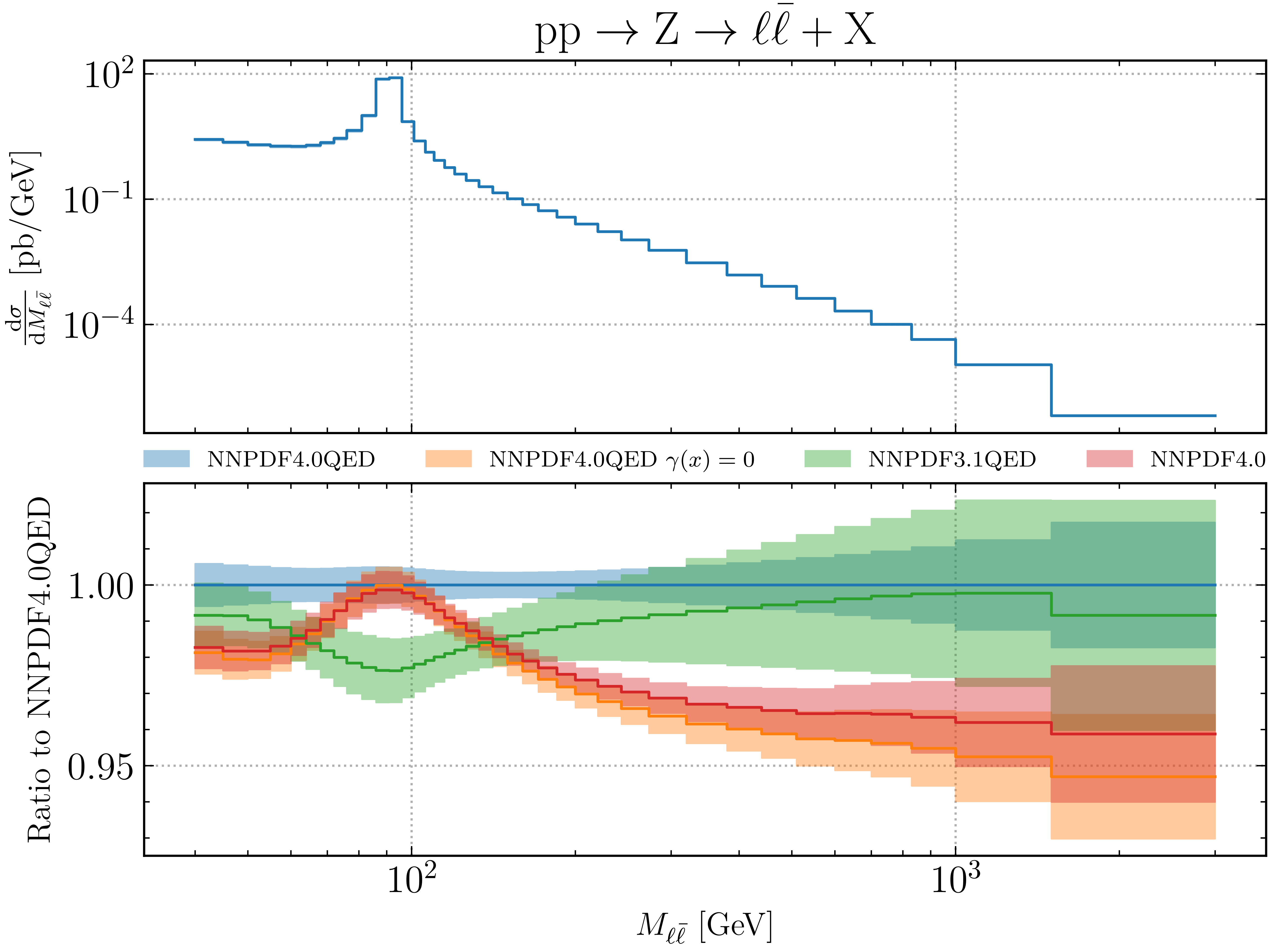
- Photon PDF:
- Difference with NNPDF3.1QED is less than percent
- Percent difference with the other photon PDFs from the latest QED fits





Impact on phenomenology

- There are regions in which QED effects are not negligible
- Difference is at the level of few percent
- Photon in subtracting momentum from the other PDFs



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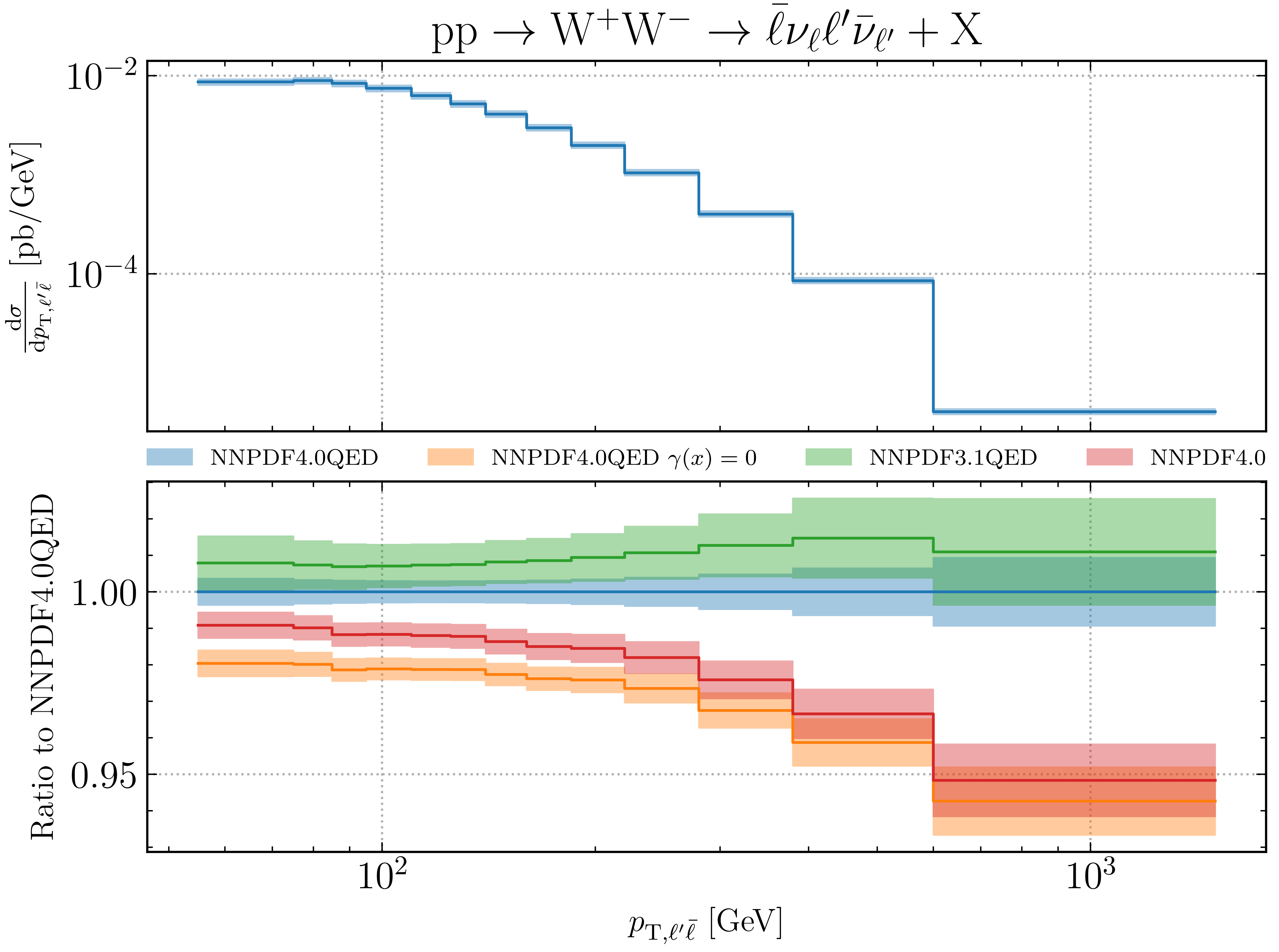

Summary and Outlook

Summary and Outlook

- ☑ We can add QED corrections to PDF fitting, getting a photon PDF
- ☑ The photon PDF is compatible with the most recent QED fits
- ☑ Quarks and gluon are almost unchanged (the photon PDF is small)
- ☑ There are processes in which photon initiated contributions are not negligible

Thank you for your attention!

Backup slides

PDFs fitting

How to add QED effects

Results

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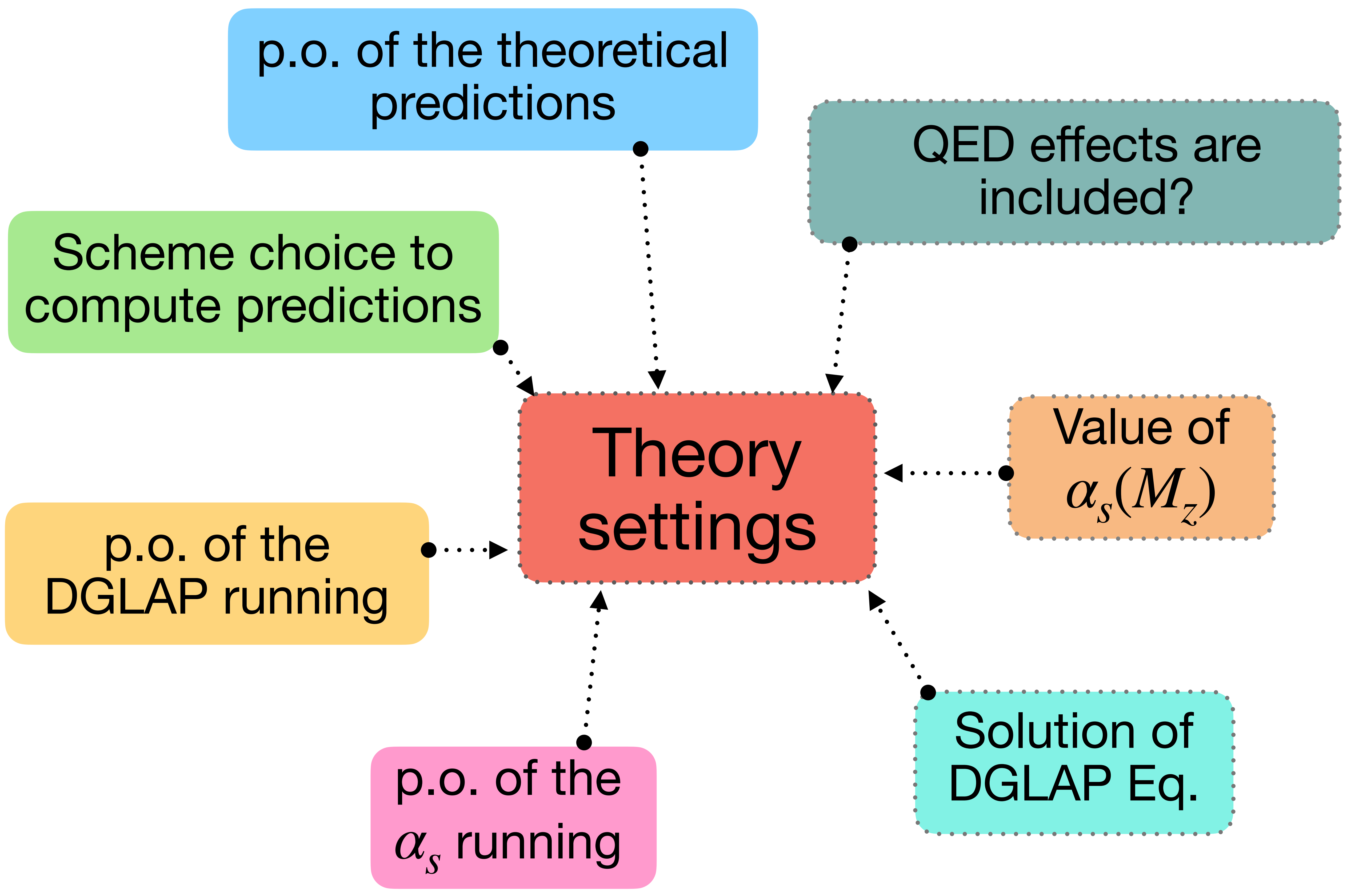
Summary and Outlook

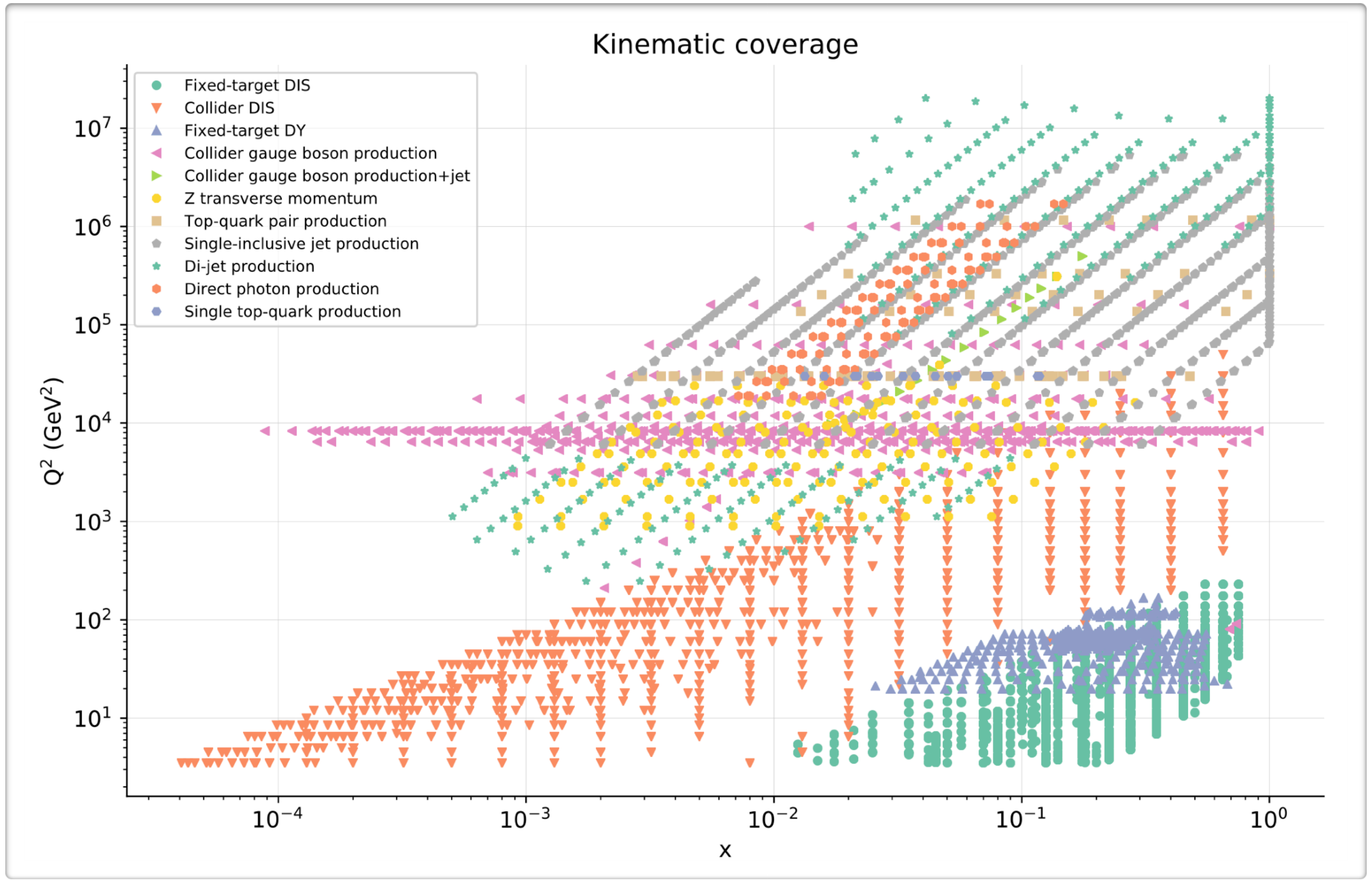
p.o.= perturbative order

- **Theory**
- What defines the theory of a fit?



- **Dataset**

- Which data points are included in the fit?



4618 data points from different processes

- **Methodology**
- How are the PDFs extracted?

PDFs are parametrized through neural networks

8 PDFs to parametrize: $g,\ u, \bar{u}, d, \bar{d}, s, \bar{s}, c$ we imposed $c = \bar{c}$

$$xf_k(x, Q_0^2; \theta) = A_k x^{1-\alpha_k} x^{\beta_k} \mathrm{NN}_k(x, \theta)$$

θ: parameters of the neural networks

Uncertainties are generated through Monte Carlo replicas

Solving DGLAP

Pure QCD case

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \Sigma \end{pmatrix} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} \otimes \begin{pmatrix} g \\ \Sigma \end{pmatrix}$$

$$\mu^2 \frac{d}{d\mu^2} V = P_{ns,v} \otimes V$$

$$\mu^2 \frac{d}{d\mu^2} f_{ns,\pm} = P_{ns,\pm} \otimes f_{ns,\pm}$$

$$\Sigma = \sum_{i=1}^{n_f} q_i^+ \quad V = \sum_{i=1}^{n_f} q_i^-$$

$$q^\pm = q \pm \bar{q}$$

$$f_{ns,\pm} = \begin{cases} u^\pm - d^\pm \\ u^\pm + d^\pm - 2s^\pm \\ u^\pm + d^\pm + s^\pm - 3c^\pm \\ u^\pm + d^\pm + s^\pm + c^\pm - 4b^\pm \\ u^\pm + d^\pm + s^\pm + c^\pm + b^\pm - 5t^\pm \end{cases}$$

QCD⊗QED case

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix} = \mathbf{P_s} \otimes \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix}$$

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} V \\ V_\Delta \end{pmatrix} = \mathbf{P_v} \otimes \begin{pmatrix} V \\ V_\Delta \end{pmatrix}$$

$$\mu^2 \frac{d}{d\mu^2} f^{u/d}_{ns,\pm} = \left(P_{ns,\pm} + \tilde{P}^{u/d}_{ns,\pm} \right) \otimes f^{u/d}_{ns,\pm}$$

$$f^u_{ns,\pm} = \begin{cases} u^\pm - c^\pm \\ u^\pm + c^\pm - 2t^\pm \end{cases} \quad f^d_{ns,\pm} = \begin{cases} d^\pm - s^\pm \\ d^\pm + s^\pm - 2b^\pm \end{cases}$$

$$\Sigma_\Delta = \frac{n_d}{n_u} \sum_{i=1}^{n_u} u_i^+ - \sum_{i=1}^{n_d} d_i^+ \quad V_\Delta = \frac{n_d}{n_u} \sum_{i=1}^{n_u} u_i^- - \sum_{i=1}^{n_d} d_i^-$$

Solving DGLAP

$$\mathbf{P_s} = \begin{pmatrix} P_{gg} + \tilde{P}_{gg} & \tilde{P}_{g\gamma} & P_{gq} + \langle \tilde{P}_{gq} \rangle & \nu_u \tilde{P}_{g\Delta q} \\ \tilde{P}_{\gamma g} & \tilde{P}_{\gamma\gamma} & \langle \tilde{P}_{\gamma q} \rangle & \nu_u \tilde{P}_{\gamma \Delta q} \\ 2n_f (P_{qg} + \langle \tilde{P}_{qg} \rangle) & 2n_f \langle \tilde{P}_{q\gamma} \rangle & P_{qq} + \langle \tilde{P}_q^{\mathrm{ns},+} \rangle + \langle e_q^2 \rangle^2 \tilde{P}_{\mathrm{ps}} & \nu_u \tilde{P}_{\Delta q}^{\mathrm{ns},+} + \nu_u e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{P}_{\mathrm{ps}} \\ 2n_f \nu_d \tilde{P}_{\Delta q g} & 2n_f \nu_d \tilde{P}_{\Delta q \gamma} & \nu_d \tilde{P}_{\Delta q}^{\mathrm{ns},+} + \nu_d e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{P}_{\mathrm{ps}} & P_{\mathrm{ns},+} + \{ \tilde{P}_q^{\mathrm{ns},+} \} + \nu_u \nu_d (e_{\Delta q}^2)^2 \tilde{P}_{\mathrm{ps}} \end{pmatrix}$$

$$\mathbf{P_v} = \begin{pmatrix} P_{\mathrm{ns},V} + \langle \tilde{P}_q^{\mathrm{ns},-} \rangle & \nu_u \tilde{P}_{\Delta q}^{\mathrm{ns},-} \\ \nu_d \tilde{P}_{\Delta q}^{\mathrm{ns},-} & P_{\mathrm{ns}-} + \{ \tilde{P}_q^{\mathrm{ns},-} \} \end{pmatrix}$$

$$\nu_{u/d} = \frac{n_{u/d}}{n_f}, \quad \langle \tilde{P}_q^{\mathrm{ns},\pm} \rangle = \nu_u \tilde{P}_u^{\mathrm{ns},\pm} + \nu_d \tilde{P}_d^{\mathrm{ns},\pm},$$

$$\{ \tilde{P}_q^{\mathrm{ns},\pm} \} = \nu_d \tilde{P}_u^{\mathrm{ns},\pm} + \nu_u \tilde{P}_d^{\mathrm{ns},\pm}, \quad \tilde{P}_{\Delta q}^{\mathrm{ns},\pm} = \tilde{P}_u^{\mathrm{ns},\pm} - \tilde{P}_d^{\mathrm{ns},\pm}$$

Solution of the non-diagonal sectors

$$\mu^{2(n)} = \mu^2$$

$$\mathbf{E}_S(\mu^2 \leftarrow \mu_0^2) = \mathscr{P} \exp\left(-\int_{\log \mu_0^2}^{\log \mu^2} \gamma_S(\alpha_s(\mu'^2), \alpha(\mu'^2)) d\log \mu'^2\right) \simeq \prod_{k=0}^{n-1} \mathbf{E}_S(\mu^{2(k+1)} \leftarrow \mu^{2(k)})$$

$$\gamma(N) = -\int_0^1 dz\, z^{N-1} P(z)$$

$$\mathbf{E}_S(\mu^{2(k+1)} \leftarrow \mu^{2(k)}) = \exp\left(-\gamma_S(\alpha_s(\mu^{2(k+1/2)}), \alpha(\mu^{2(k+1/2)}))\Delta \log \mu^{2(k)}\right)$$

Solved in Mellin space

$$\log \mu^{2(k+1/2)} = \frac{\log \mu^{2(k+1)} + \log \mu^{2(k)}}{2}$$

$$\Delta \log \mu^{2(k)} = \log \mu^{2(k+1)} - \log \mu^{2(k)}$$

Computation of the photon

Why the LuxQED formula is used at 100 GeV?

LuxQED neglects higher twist corrections $\mathcal{O}\left(\frac{\Lambda}{\mu}\right)$

For low μ, the integral is dominated by low Q^2 structure functions **non-perturbative!**